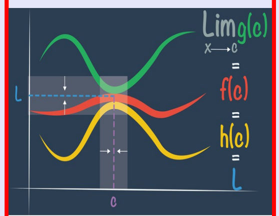
Calculus I

Lecture 20



Feb 19-8:47 AM

Class QZ 15

Verify all Conditions of the Rolle's theorem for $f(x) = 3x^2 - 12x + 5$ on $[1,3]$ then find all numbers c that satisfy the Conclusion of the Rolle's Theorem.

$f(x) = 3x^2 - 12x + 5$, $[1,3]$

Polynomial function $\Rightarrow$ Continuous and diff. $(-\infty,\infty)$

$f(1) = 3(1)^2 - 12(1) + 5 = -4$

$f(3) = 3(3)^2 - 12(3) + 5 = -4$

1) Cont. on $[1,3]$ ✓

2) diff. on $(1,3)$ ✓

3) $f(1) = f(3)$ ✓

$f'(c) = 0 \rightarrow f'(x) = 6x - 12$

$f'(c) = 6c - 12 = 0 \rightarrow \boxed{c=2}$ is in $(1,3)$

Jul 21-7:21 AM

Graph $f(x)=x^2-6x$ on $[-1,6]$

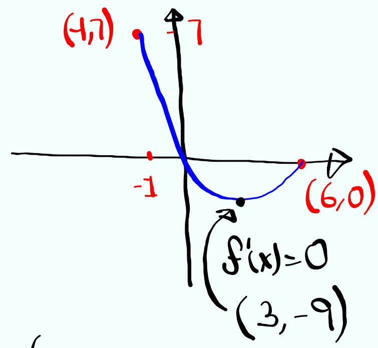
$f(3)=3^2-6(3)$
$=-9$

$f(-1)=(-1)^2-6(-1)=7$

$f(6)=6^2-6(6)=0$

$f(x)$ is Cont $[-1,6]$

$f(x)$ is diff. $(-1,6)$



$f(x)=x^2-6x \qquad f'(x)=2x-6$

$2x-6=0 \qquad \boxed{x=3}$

Abs. Max at $(-1,7)$ Max. Value 7

Abs. Min. at $(3,-9)$ Min Value -9

Jul 21-8:18 AM

Find Abs. Max & Abs. Min for

$f(x)=12+4x-x^2$ over $[0,5]$.

$f(0)=12$

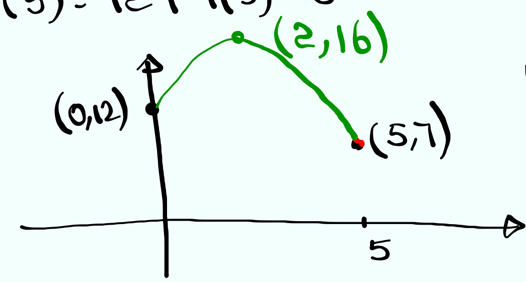
$f(5)=12+4(5)-5^2=7$

$f(x)$ is Cont. & diff.

$f'(x)=4-2x$

$f'(x)=0 \qquad x=2$

$f(2)=16$



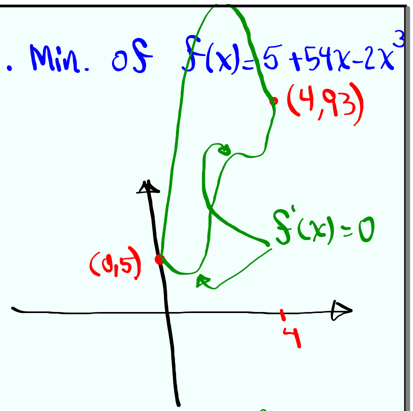
Max. Value at $(2,16)$

Min. Value at $(5,7)$

Max. Value 16, Min. Value 7

Jul 21-8:23 AM

Find Abs. Max & Abs. Min. of $f(x)=5+54x-2x^3$ on $[0,4]$.



$f(0)=5$

$f(4)=5+54(4)-2(4)^3=93$

$f(x)$ is polynomial $\rightarrow$ Cont. & diff.

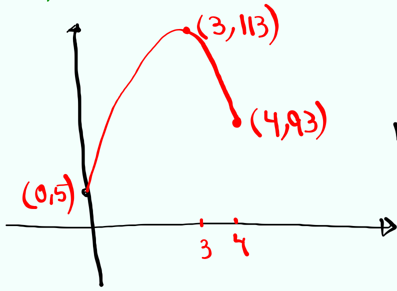
$f(3)=5+54(3)-2(3)^3=113$

$f'(x)=54-6x^2$

$54-6x^2=0$

$9-x^2=0 \quad x=3$

$x=-3$



Max Value of 113 at $x=3$

Min Value of 5 at $x=0$

Jul 21-8:30 AM

Find Abs. Max & Abs. Min. for $f(x)=x\sqrt{4-x^2}$ on $[-1,2]$.

Domain

$4-x^2\geq 0$

$-x^2\geq -4$

$x^2\leq 4$

$-2\leq x\leq 2$

$f(x)$ is cont. on $[-1,2]$

$f(-1)=-1\sqrt{4-(-1)^2}=-\sqrt{3}$

$f(2)=2\sqrt{4-2^2}=0$

$f(x)=x(4-x^2)^{1/2}$

$f'(x)=1(4-x^2)^{1/2}+x\cdot\frac{1}{\cancel{2}}(4-x^2)^{-1/2}\cdot(-\cancel{2}x)$

$=(4-x^2)^{1/2}-x^2(4-x^2)^{-1/2}$

$=(4-x^2)^{-1/2}(4-x^2-x^2)=\dfrac{4-2x^2}{\sqrt{4-x^2}}$

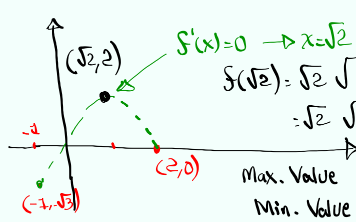
$f'(x)=0 \quad 4-2x^2=0$

$x^2=2$

$x=\pm\sqrt{2}$

$f'(x)$ undefined

$4-x^2=0 \quad x=\pm 2$



$f'(x)=0 \rightarrow x=\sqrt{2}$

$f(\sqrt{2})=\sqrt{2}\sqrt{4-(\sqrt{2})^2}$

$=\sqrt{2}\sqrt{4-2}=\sqrt{2}\sqrt{2}=\boxed{2}$

Max. Value of 2 at $x=\sqrt{2}$

Min. Value of $-\sqrt{3}$ at $x=-1$

Jul 21-8:40 AM

$f(x) = \dfrac{1}{x^2-9}$

1) Domain $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$

$x^2-9 \neq 0$

$x \neq \pm 3$

2) V.A. $x = \pm 3$

3) All intercepts

x-Int. None

y-Int $(0, -\frac{1}{9})$

4) H.A.

$\lim\limits_{x \to \pm\infty} \dfrac{1}{x^2-9} = 0$

$y = 0$

5) Determine even, odd, or neither function.

$f(-x) = \dfrac{1}{(-x)^2-9} = \dfrac{1}{x^2-9} = f(x)$ even function

Symmetric w/t y-axis.

6) $f'(x)$ & $f''(x)$

$f'(x) = \dfrac{-2x}{(x^2-9)^2}$

$f''(x) = \dfrac{6(x^2+3)}{(x^2-9)^3}$

$f'(x) = 0 \to x = 0$

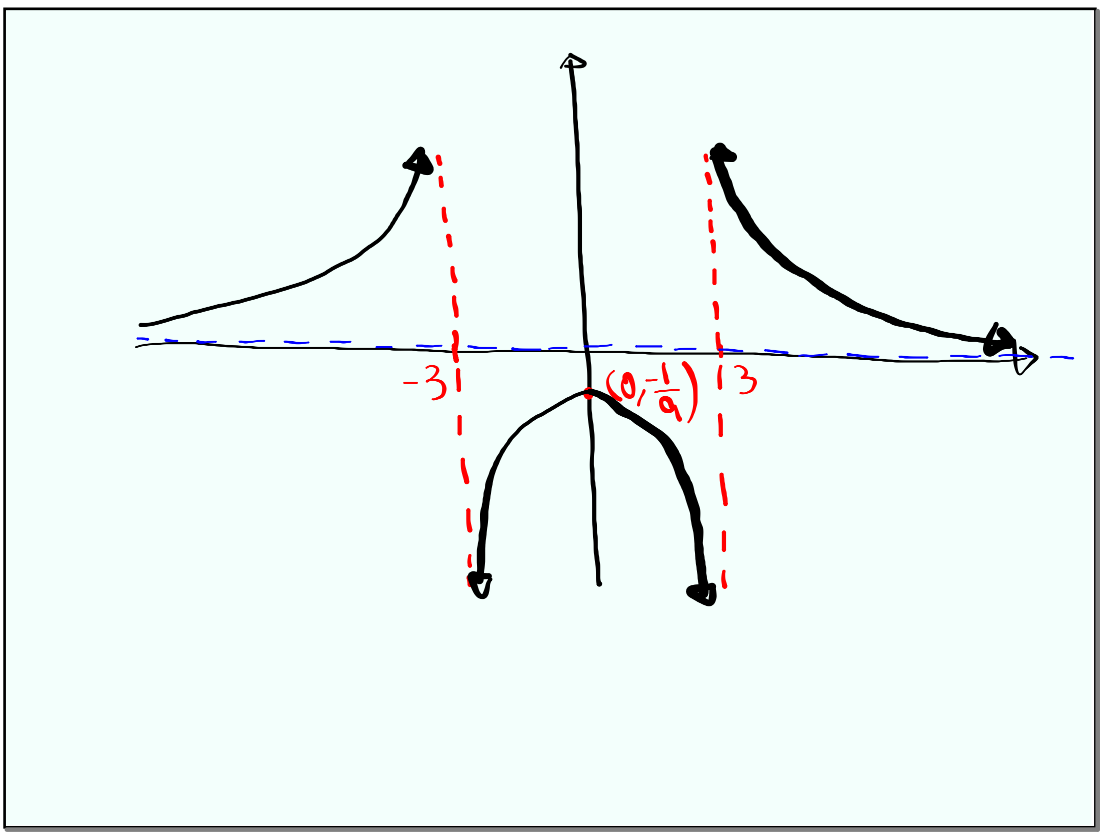
$f'(x)$ und. at $x = \pm 3$

$f''(x) \neq 0$

$f''(x)$ und. at $x = \pm 3$

x	$-\infty$		-3		0		3		∞
$f'(x)$		+	○	+	●	−	○	−	
$f''(x)$		+	○	−		−	○	+	
$f(x)$									

Jul 21-8:55 AM



Jul 21-9:13 AM

x & y $\qquad x-y=100$

Find two numbers whose difference is 100 $\quad \to y = x-100$

and their product is minimum.

xy is minimum.

$x(x-100)$ is Min.

$$f(x) = x(x-100) = x^2 - 100x$$

$$f'(x)=0 \qquad 2x-100=0 \qquad \boxed{x=50}$$

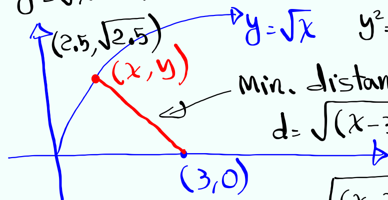
$$y = 50-100$$

$$\boxed{y=-50}$$

50 & −50

Jul 21-9:15 AM

Find the point of the graph of $y=\sqrt{x}$ that is the closest to $(3,0)$.



$y^2 = x$

Min. distance

$$d = \sqrt{(x-3)^2 + (y-0)^2}$$

$$= \sqrt{(x-3)^2 + y^2}$$

$$= \sqrt{(x-3)^2 + x}$$

To minimize the distance, we must minimize the radicand

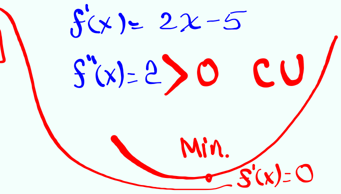
$$f(x) = (x-3)^2 + x \qquad f'(x) = 2(x-3)^1 \cdot 1 + 1$$

$$f'(x) = 2x - 6 + 1$$

$$2x-5=0 \leftarrow f'(x) = 2x-5$$

$$\boxed{x=\frac{5}{2}} \quad \boxed{x=2.5}$$

$$f''(x) = 2 > 0 \quad \text{CU}$$



Jul 21-9:21 AM

Introduction to integration

Reverse of differentiation

$\int \ldots dx$ with respect to x.

integral

Integrand

$$\int f'(x)\,dx = f(x) + C$$

$$\frac{d}{dx}[\text{Answer}] = \text{integrand}$$

ex $\int 2x\,dx = x^2 + C$

$\frac{d}{dx}[x^2 + C] = 2x$

ex: $\int \cos x\,dx = \sin x + C$

$\frac{d}{dx}[\sin x + C] = \cos x$

Jul 21-9:46 AM

Some Rules:

1) $\int c\, f(x)dx = c\int f(x)dx$

2) $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$

3) $\int k\,dx = kx + C$

4) $\boxed{\int x^n dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1}$

ex: $\int (x^3 + 8)dx = \int x^3 dx + \int 8\,dx = \frac{x^{3+1}}{3+1} + 8x + C$

$= \boxed{\frac{x^4}{4} + 8x + C}$

$\frac{d}{dx}\left[\frac{x^4}{4} + 8x + C\right] = \frac{\cancel{4}}{\cancel{4}}x^3 + 8 = \boxed{x^3 + 8}$

$\int (\cos x - \sqrt{x})dx = \int \cos x\,dx - \int x^{\frac{1}{2}}dx$

$= \sin x - \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \sin x - \frac{x^{3/2}}{3/2} + C$

$= \boxed{\sin x - \frac{2}{3}x^{\frac{3}{2}} + C}$

$\frac{d}{dx}\left[\sin x - \frac{2}{3}x^{\frac{3}{2}} + C\right] =$

$\cos x - \frac{\cancel{2}}{\cancel{3}}\cdot\frac{\cancel{3}}{\cancel{2}}x^{\frac{3}{2}-1} = \cos x - x^{\frac{1}{2}} = \boxed{\cos x - \sqrt{x}}$

Jul 21-9:52 AM

Evaluate

1) $\int \left(x^2 + \frac{1}{x^2}\right) dx = \int x^2 dx + \int x^{-2} dx$

$= \frac{x^{2+1}}{2+1} + \frac{x^{-2+1}}{-2+1} + C$

$= \frac{1}{3}x^3 - x^{-1} + C$

$= \frac{1}{3}x^3 - \frac{1}{x} + C$

Jul 21-10:05 AM

Integral of Basic Trig functions:

1) $\int \sin x \, dx = -\cos x + C$

2) $\int \cos x \, dx = \sin x + C$

3) $\int \sec^2 x \, dx = \tan x + C$

4) $\int \csc^2 x \, dx = -\cot x + C$

5) $\int \sec x \tan x \, dx = \sec x + C$

6) $\int \csc x \cot x \, dx = -\csc x + C$

Jul 21-10:10 AM

Evaluate

1) $\int \sec x(\sec x + \tan x)dx$

$= \int [\sec^2 x + \sec x \tan x]dx = \boxed{\tan x + \sec x + C}$

2) $\int \frac{\sin 2x}{\sin x} dx = \int \frac{2\cancel{\sin x}\cos x}{\cancel{\sin x}} dx = \int 2\cos x\, dx$

$= \boxed{2\sin x + C}$

Jul 21-10:15 AM

Defenite Integral

$\int_a^b f'(x)dx = f(x)\Big|_a^b = f(b) - f(a)$

$\int_1^2 2x\, dx = x^2\Big|_1^2 = 2^2 - 1^2 = 4 - 1 = \boxed{3}$

Evaluate $\int_0^2 [3x^2 + 1]dx = (x^3 + x)\Big|_0^2$

$= (2^3 + 2) - (0^3 + 0) = \boxed{10}$

Evaluate $\int_0^{\pi} \sin x\, dx = -\cos x\Big|_0^{\pi} = -(\cos\pi - \cos 0)$

$= -(-1 - 1)$

$= -(-2)$

$= \boxed{2}$

Jul 21-10:21 AM

Evaluate

1) $\int_0^{\pi/2} \cos x\,dx$

$= \sin x \Big|_0^{\pi/2}$

$= \cancel{\sin \frac{\pi}{2}} \to 1 - \cancel{\sin 0} \to 0$

$= \boxed{1}$

2) $\int_{\frac{\pi}{4}}^{\pi/3} \sec^2 x\,dx$

$= \tan x \Big|_{\pi/4}^{\pi/3}$

$= \tan \frac{\pi}{3} - \tan \frac{\pi}{4}$

$= \boxed{\sqrt{3} - 1}$

3) $\int_4^9 \sqrt{x}\,dx = \int_4^9 x^{1/2}dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\Big|_4^9$

$= \frac{x^{3/2}}{3/2}\Big|_4^9 = \frac{2}{3}x\sqrt{x}\Big|_4^9$

$= \frac{2}{3}\left[9\sqrt{9} - 4\sqrt{4}\right]$

$= \frac{2}{3}\left[27 - 8\right] = \frac{2}{3}\cdot 19 = \boxed{\frac{38}{3}}$

Jul 21-10:29 AM

$\int_1^2 (4x^3 - 3x^2 + 2x)\,dx$

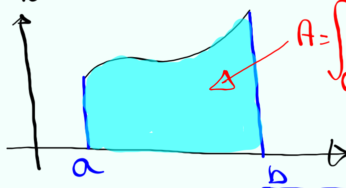
$= (x^4 - x^3 + x^2)\Big|_1^2$

$= \left[2^4 - 2^3 + 2^2\right] - \left[1^4 - 1^3 + 1^2\right]$

$= [\underbrace{16 - \cancel{8}}_{8} \underbrace{+\cancel{4}}_{12}] - [\cancel{1} - \cancel{1} + 1] = 12 - 1 = \boxed{11}$

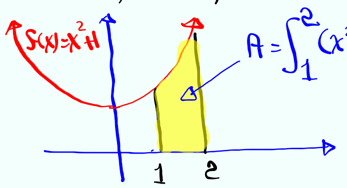
Jul 21-10:41 AM

If $f(x) \geq 0$ and cont. on $[a,b]$, then the area bounded by $f(x)$, x-axis, $x=a$, and $x=b$



$A = \int_a^b f(x)\,dx$

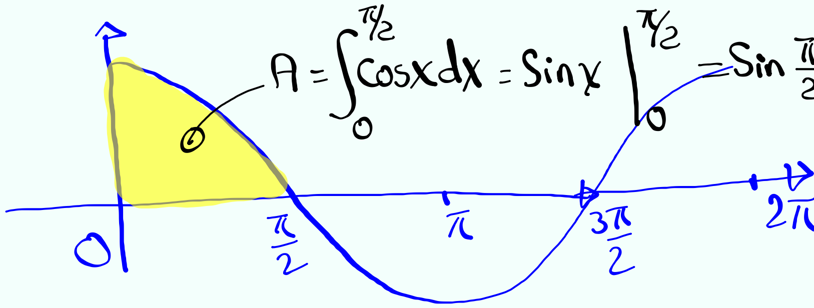
Find the area bounded by $\boxed{f(x) = x^2+1}$, x-axis, $x=1$, and $x=2$.



$$A = \int_1^2 (x^2+1)\,dx = \left(\frac{x^3}{3} + x\right)\Bigg|_1^2$$
$$= \left(\frac{2^3}{3} + 2\right) - \left(\frac{1^3}{3} + 1\right)$$
$$= \frac{8}{3} + 2 - \frac{1}{3} - 1$$
$$= 1 + \frac{7}{3} = \boxed{\frac{10}{3}}$$

Jul 21-10:46 AM

Find the area below $\boxed{f(x) = \cos x}$, above x-axis from $x=0$ to $x=\frac{\pi}{2}$.

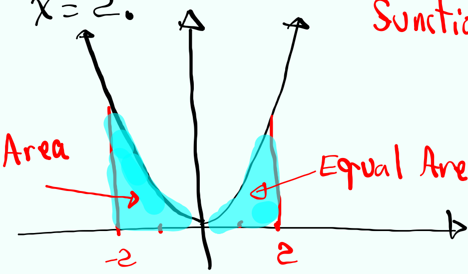


$$A = \int_0^{\pi/2} \cos x\,dx = \sin x \Bigg|_0^{\pi/2} = \sin\frac{\pi}{2} - \sin 0$$
$$= \boxed{1}$$

Jul 21-10:54 AM

Find the area below $f(x) = x^4$, above x-axis from $x = -2$ to $x = 2$.

even function



$$A = 2\int_0^2 x^4\,dx = 2 \cdot \frac{x^5}{5}\Big|_0^2 = \frac{2}{5}\left(2^5 - 0^5\right) = \boxed{\frac{64}{5}}$$

$$A = \int_{-2}^{2} x^4\,dx = \frac{x^5}{5}\Big|_{-2}^{2} = \frac{1}{5}\left[2^5 - (-2)^5\right]$$

$$= \frac{1}{5}\left[32 + 32\right] = \boxed{\frac{64}{5}}$$

Jul 21-10:59 AM